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# CLOSED-FORM SOLUTION TO GUIDANCE PROBLEMS

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## ABSTRACT

The effect of inertia of the guidance systems on the closed-form solution to guidance problems is investigated and an analytical solution is obtained. It indicates that the solution without considering the inertia is only a special form for long Line-of-Sight, However, this solution is also available to short Line-of-Sight. The unstability of angular rate of the Line-of-Sight and the reason for miss-distances are also expressed by this solution. The results can be used to analyse the effect of the proportional-navigation coefficient and the inertia of guidance system on miss-distances.

**Key words**  guidance,  closed-form solution to guidance problem,  proportional navigation, miss distance

There is a long history to the closed-form solution to guidance problems [1-7]. The application of research used in mechanics is an important means in evaluating guidance laws, guidance parameters, and attack conditions as affecting missile interception properties. However, without a single exception the research in cited references neglects the time lag between the command acceleration $A_c$ and actual acceleration A. If $1/(\tau p+1)$ is the simplest description of the time lag (p is the differential operator) and $t_{q0}$ indicates the time-to-go. With respect to $\tau \ll t_{q0}$ (long apparent distances), their research

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results on zero off-target attack conditions and the value range of the guidance parameters are correct. However, for $\tau > t_{go}$ (short apparent distance) their conclusions are incorrect.

In their research, the solution to the time-lag guidance problems is considered (mainly for short apparent distances) as their is no great difference between the long apparent distance and the available conclusions). The analytical conclusion is closer to practice.

## 1. Mathematical Model of Guidance Problems

Let us assume that the problem to be discussed is limited to planar attack indicated in Fig. 1. In the figure, these symbols indicate the following: T is target; I is interceptor; Txyz is the horizontally shifted coordinate; $\mathbf{R}=r\mathbf{u}_r$, $\mathbf{V}=v\mathbf{u}_v$, $\mathbf{A}=a\mathbf{u}_a$, $\Omega=\omega\mathbf{u}_\omega$ respectively, are line-of-sight distance, relative velocity, relative acceleration, and vector of rotational angular velocity of line-of-sight. $r$, $\mathbf{u}_r$, $v$, $\mathbf{u}_v$, $a$, $\mathbf{u}_a$, $\omega$, $\mathbf{u}_\omega$ correspond to their modes and unit vectors. Based on common sense, it is not difficult to derive the following equations:

$$\mathbf{R} = \dot{\mathbf{V}} = \mathbf{A} \qquad\qquad (1)$$

$$\Omega = \mathbf{R} \times \mathbf{V} / r^2 \qquad\qquad (2)$$

$$\mathbf{V} = \dot{r}\mathbf{u}_r + \Omega \times \mathbf{R} \qquad\qquad (3)$$

By differentiating Eq. (3), we have

$$\mathbf{A} = r\mathbf{u}_r + \Omega \times (\Omega \times \mathbf{R}) + 2\Omega \times \dot{r}\mathbf{u}_r + \dot{\Omega} \times \mathbf{R} \qquad (4)$$

With respect to Eq. (4), let us derive the dot product and cross product of R. It is assumed that only the actual proportional guidance problem (orthogonal intersection between A and R) is discussed, then we obtain the two following equations:
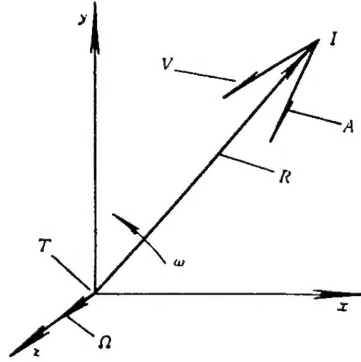
Fig. 1. Planar attack

$$\ddot{r} = r\omega^2 + \mathbf{R} \cdot \mathbf{A}_t; \qquad \begin{cases} \dot{r}(0) = \dot{r}_0 \\ r(0) = r_0 \end{cases} \tag{5}$$

$$\mathbf{R} \times \mathbf{A}_i - \mathbf{R} \times \mathbf{A}_t = 2r\dot{r}\Omega + r^2\dot{\Omega} \tag{6}$$

In Eq. (6) $\mathbf{A}_i$ and $\mathbf{A}_t$ indicate, respectively, the acceleration vectors of the interceptor and target; $\mathbf{A} = \mathbf{A}_i - \mathbf{A}_t$. If $\mathbf{A}_{ic}$ is the command acceleration, then $\mathbf{A}_i = \mathbf{A}_{ic}/(\tau p + 1)$. For actual proportional guidance, $\mathbf{A}_{ic} = -K_\omega \dot{r}\mathbf{u}_r \times \Omega$, $K_\omega$ is the proportional-guidance constant. By substituting $\mathbf{A}_{ic}$ and $\mathbf{A}_i$ in Eq. (6), we obtain:

$$\tau r\ddot{\Omega} + (4\tau\dot{r} + r)\dot{\Omega} + \left(2\tau\ddot{r} + \frac{2\tau\dot{r}^2}{r} + 2\dot{r} - k_\omega\dot{r}\right)\Omega$$

$$= \tau\dot{\mathbf{R}} \times \mathbf{A}_t + \tau\mathbf{R} \times \dot{\mathbf{A}}_t + \mathbf{R} \times \mathbf{A}_t; \qquad \begin{cases} \dot{\Omega}(0) = \dot{\Omega}_0 \\ \Omega(0) = \Omega_0 \end{cases} \tag{7}$$

Eqs. (5) and (7) constitute a mathematical model of the guidance problem with consideration of command time lag.

2. Simplification of Mathematical Model

Let us assume that the target flies along a straight line at constant velocity ( $\mathbf{A}_t = \dot{\mathbf{A}}_t = 0$ ). It is also assumed that the solution to guidance problems is herein discussed by using long and short line-of-sight distances, respectively, and thus Eqs.

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(5) and (7) can be simplified.

For long line-of-sight distances, $\tau \ll t_{go}$, $\tau \approx 0$ , Eqs. (5) and (7) become

$$\ddot{r} - r\omega^2 = 0; \qquad \dot{r}(0) = \dot{r}_0, \ r(0) = r_0 \tag{8}$$

$$\dot{\Omega} + (2 - K_\omega)\frac{\dot{r}}{r}\Omega = 0; \qquad \Omega(0) = \Omega_0 \tag{9}$$

Solutions to Eqs. (8) and (9) should be close to the solutions of Eqs. (5) and (7) with consideration of time lag. However, solutions to Eqs. (8) and (9) have been given in reference [1].

For short line-of-sight distances, when r is small and not equal to 0, Eq. (7) can be simplified:

$$\ddot{\Omega} + 4\frac{\dot{r}}{r}\dot{\Omega} + 2\left(\frac{\dot{r}}{r}\right)^2\Omega = 0; \qquad \dot{\Omega}(0) = \Omega_0, \ \Omega(0) = \Omega_0 \tag{10}$$

From the following analysis, we can see that there are considerable differences between the solutions to Eqs. (8) and (10) and the solutions to Eqs. (8) and (9). Therefore, the solutions in reference [1] are not adaptable to short line-of-sight distances.

3. Closed-Form Solution of Guidance Problems with Short Line-of-Sight Distances

Define $d\zeta / dt = \dot{r} / r (r \neq 0)$ or $\zeta = \ln(r / r_0)$ . $r_0$ is the initial value of the short line-of-sight distance. Thus $d\Omega / dt = (\dot{r} / r)(d\Omega / d\zeta)$ and $d^2\Omega / dt^2 = (\dot{r} / r)^2(d^2\Omega / d\zeta^2)$ . By substituting in Eq. (10), we obtain

$$\frac{d^2\Omega}{d\zeta^2} + 4\frac{d\Omega}{d\zeta} + 2\Omega = 0; \qquad \dot{\Omega}(0) = \dot{\Omega}_0, \ \Omega(0) = \Omega_0 \tag{11}$$

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If $\dot{\Omega}(0) = 0$ , solve for Eq. (11)

$$\Omega(\zeta) = \Omega_0(1.21e^{-0.59\zeta} - 0.21e^{-3.41\zeta}) \qquad (12)$$

By substituting $\zeta = \ln(r/r_0)$ into Eq. (12) and by assuming that $\rho = r/r_0$ , we obtain

$$\Omega(\rho) = \Omega_0(1.21\rho^{-0.59} - 0.21\rho^{-3.41}) = \Omega_0\xi(\rho) \qquad (13)$$

In the equation, $\xi(\rho) = 1.21\rho^{-0.59} - 0.21\rho^{-3.41}$ , when r-->0, then $\xi(\rho) \to \infty$ $\Omega(\rho)$ is divergent.

By dividing Eq. (8) by $r_0$ and by letting $\rho = r/r_0$ and $\dot{\rho} = \dot{r}/r_0$, Eq. (8) becomes

$$\rho = \rho\omega^2; \qquad \dot{\rho}(0) = \dot{\rho}_0, \qquad \rho = \rho_0 \qquad (14)$$

In the equation, $\omega^2 = |\Omega(\rho)|^2 = \omega_0^2\xi^2(\rho),$ , and by multiplying Eq. (14) with $\dot{\rho}$ and then integrating, we obtain

$$\dot{\rho}^2 - \dot{\rho}_0^2 = 2\int_0^t \rho\dot{\rho}\,dt = 2\omega_0^2\eta(\rho) \qquad (15)$$

In the equation, $\eta(\rho)$ is

$$\eta(\rho) = 1.82\rho^{0.82} + 0.27\rho^{-2} - 0.01\rho^{-4.82} - 2.08 \qquad (16)$$

Dividing Eq. (15) with $\dot{\rho}_0^2$ and by letting $\alpha = (\omega_0/\dot{\rho}_0)^2$ , then Eq. (15) becomes $\dot{\rho}^2/\dot{\rho}_0^2 = 2\alpha\eta(\rho) + 1,$ then by substituting $\dot{\rho} = \dot{r}/r_0$ and $\dot{\rho}_0 = \dot{r}_0/r$ , we obtain

$$\dot{r} = \dot{r}_0\sqrt{2\alpha\eta(\rho) + 1} \qquad (17)$$

By dividing Eq. (17) with $r_0$, and then multiplying with dt before integration, we obtain

$$t = \frac{1}{\dot{\rho}_0}\int_1^\rho \frac{1}{\sqrt{2\alpha\eta(\rho) + 1}}\,d\rho \qquad (18)$$

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In Eq. (18), $\dot{\rho}_0$, $\alpha$ are known; and $\eta(\rho)$ and $\rho$ are known functions of r. Therefore, t can be derived with the given r. Then we can obtain $\rho(t)$ (or r(t)). By substituting $\rho(t)$ into Eq. (13), we can derive $\Omega(t)$ (that is, $\omega(t)$). r(t) and $\omega(t)$ are solutions to the guidance problem. This is to say, that Eqs. (13) and (18) are closed-form solutions of short line-of-sight distances in guidance problems.

## 4. Discussion

(1) Determination of the range of short line-of-sight distances

The initial value $r_0$ of the short line-of-slight distance is determined by the instability critical conditions of the system. From Eq. (7), we know that $4\tau\dot{r}_0 + r_0 = 0$ . By solving we obtain

$$r_0 = -4\tau\dot{r}_0 \qquad (19)$$

which equation is related to $\tau$ and $r_0$. The high-speed interceptor $r_0$ is approximately longer than a kilometer, and a low-speed interceptor is approximately tens of meters.

(2) Solutions to guidance problems with blind zone of guidance head

In the guidance zone blind head the missile is without control. $K_\theta = 0$ is the case of short line-of-sight distances. Therefore, the mathematical model is the same as that of Eqs. (8) and (10), therefore the solutions are also the same. In both cases (with or without control) of short line-of-sight distances, $\omega(t)$ are unstable (r=0).

(3) The effect on the solutions with limited acceleration in the acceptor.

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If the interceptor acceleration is limited, in Eqs. (9) the proportional guidance constant $K_\omega$ is decreased. When $K_\omega < 2$, (t) becomes unstable in the entire range domain of line-of-sight distances.

(4) Oscillatory mathematical model of $\omega(t)$ and its solution

The nonsimplified Eq. (7) is the oscillator mathematical model of $\omega(t)$. This is adaptable to the line-of-sight range of r approximately equal to $r_0$. This is difficult to see an analytical solution for it but a numerical solution can be derived.

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## REFERENCES

1 Cochran J E Jr, Ho T S, Thaxton D G. Analytical Solutions to a Guidance Problem. J Guidance, 1991, 14(1): 117 ~122

2 Yang C-D, Yeb F-B, Chen J-H. The Closed-Form Solution of Generalized Proportional Navigation. J Guidance, 1987, 10(2): 216~218

3 Yang C-D, Yeb F-B, Chen J-H. The Closed-Form Solution for a Class of Guidance Laws. J Guidance. 1987, 10(4): 412~415

4 Guelman M. The Closed-Form Solution of True Proportional Navigation. IEEE Transactions on Aerospace and Electronic System, 1976, AES(12): 472~480

5 Murtagh S A, Griel H E. Fundamentals of Proportional Navigation. IEEE Spectrums, 1966. 75~85

6 Gullman M. A Qualitative Study of Proportional Navigation. IEEE Transactions on Aerospace and Electronic System. 1972, AES(7): 637~643

7 Adier F. Missile Guidance by Three-dimensional Proportional Navigation. J Applied Physics, 1965, 500~507

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